

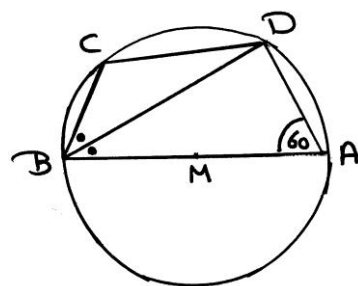
Final revision**[1] In the opposite figure**

ABCD is a cyclic quadrilateral in which :

\overline{AB} is a diameter and \overline{BD} bisects $\angle ABC$

If $m(\angle A) = 60^\circ$

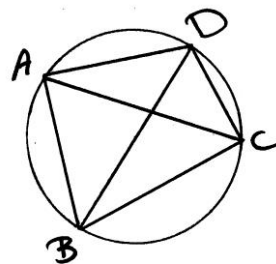
Find with proof : $m(\angle CDB)$

**[2] In the opposite figure**

ABCD is a cyclic quadrilateral such that :

$AC = BD$

Proof that : $AB = CD$

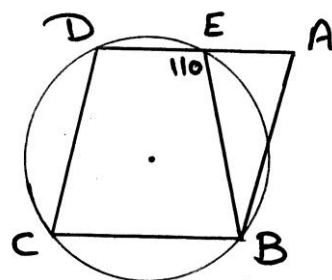
**[3] In the opposite figure**

ABCD is a parallelogram

$E \in \overline{AD}$, $m(\angle BED) = 110^\circ$

(i) Proof that : $AB = EB$

(ii) Find : $m(\angle ABE)$, $m(\angle EBC)$

**[4] In the opposite figure**

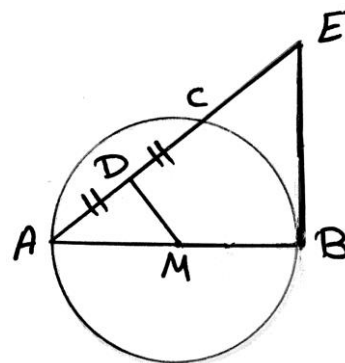
\overline{AB} is a diameter in the circle M.

\overline{BE} is a tangent to the circle at B

D is the midpoint of \overline{AC}

Proof that : (i) MBED is a cyclic quad.

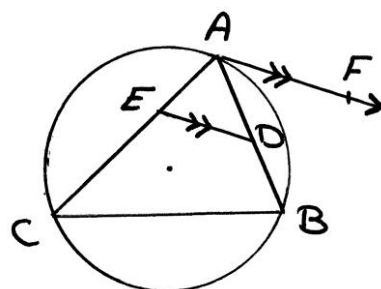
(ii) $\overline{MD} \parallel \overline{BC}$

**[5] In the opposite figure**

\overline{FA} is a tangent to the circle at A

$\overline{DE} \parallel \overline{FA}$

Proof that : DBCE is a cyclic quad.



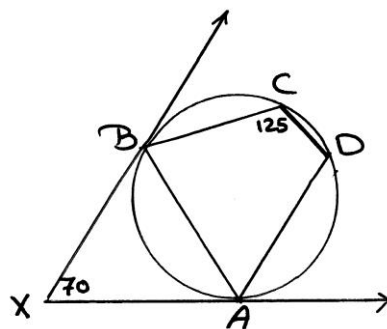
[6] In the opposite figure

\overrightarrow{XA} and \overrightarrow{XB} are two tangents to the circle at A and B

$$m(\angle AXB) = 70^\circ, m(\angle DCB) = 125^\circ$$

Proof that : (i) \overrightarrow{AB} bisects $\angle DAX$

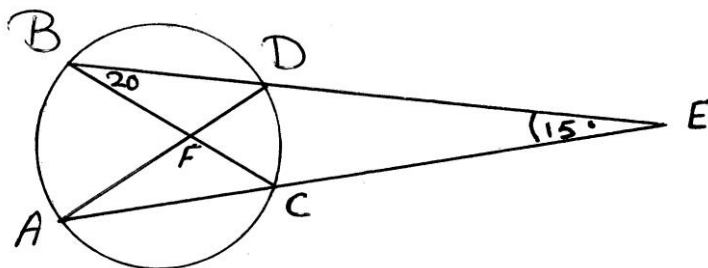
$$(ii) \overrightarrow{AD} \parallel \overrightarrow{XB}$$

**[7] In the opposite figure**

$$m(\angle DBC) = 20^\circ$$

$$, m(\angle E) = 15^\circ$$

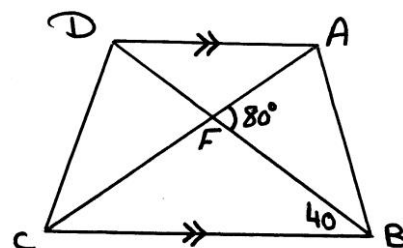
Find : $m(\angle AFB)$

**[8] In the opposite figure**

$$m(\angle AFB) = 80^\circ, m(\angle DBC) = 40^\circ$$

$$, \overrightarrow{AD} \parallel \overrightarrow{BC}$$

Proof that : ABCD is a cyclic quad.

**[9] In the opposite figure**

\overline{AC} is a diameter in the circle M.

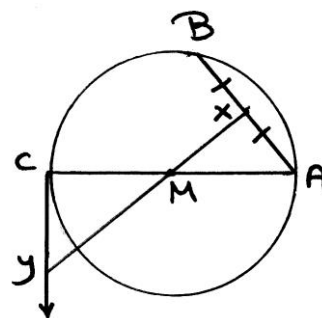
X is the midpoint of \overline{AB}

\overrightarrow{CY} is a tangent to the circle at C

and cuts \overrightarrow{XM} at Y

Proof that : (i) AXCY is a cyclic quad.

$$(ii) m(\angle BMC) = 2m(\angle MYC)$$

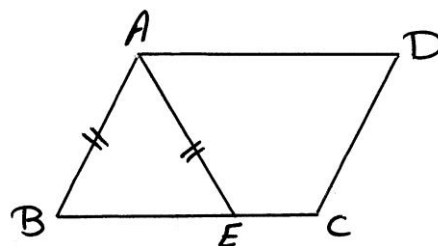
**[10] In the opposite figure**

ABCD is a parallelogram

$E \in \overline{BC}$ such that : $AB = AE$

Proof that : (i) AECD is a cyclic quad.

$$(ii) \overrightarrow{AD} \text{ is a tangent to the circumcircle of } \triangle ABE$$



The diagram shows a circle inscribed in a triangle XYZ . The circle is tangent to the sides XY and XZ at points Y and Z respectively. A point D is located on the circle, and a line segment DE is drawn, where E is another point on the circle. The angle $\angle X$ is labeled as 80° , and the angle $\angle ZDE$ is labeled as 130° .

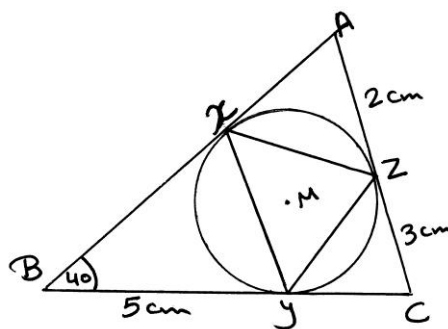
[16] In the opposite figure

M is a circle touching the sides of the triangle ABC at X, Y and Z

If $AX = 2\text{cm}$, $YB = 5\text{cm}$,

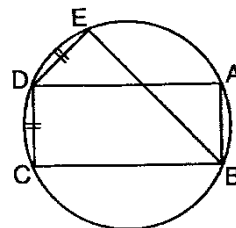
$CZ = 3\text{cm}$, $m(\angle B) = 40^\circ$

Find : (i) the perimeter of $\triangle ABC$
(ii) $m(\angle XZY)$

**[17] In the opposite figure**

ABCD is a rectangle inscribed in a circle and $DE = DC$

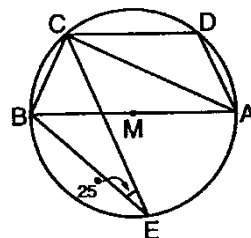
Prove that : $AD = BE$

**[18] In the opposite figure**

\overline{AB} is a diameter and

$m(\angle BEC) = 25^\circ$

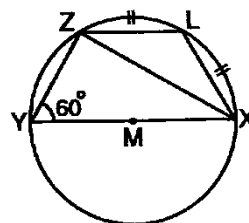
Find : $m(\angle BAC)$, $m(\angle ABC)$ and $m(\angle ADC)$

**[19] In the opposite figure**

\overline{XY} is a diameter in circle M,

$m(\angle XL) = m(\angle LZ)$ and $m(\angle Y) = 60^\circ$

Find : $m(\angle L)$, $m(\angle XZY)$ and $m(\angle LXZ)$

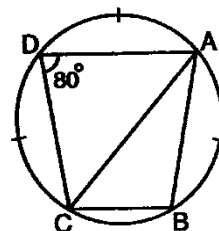
**[20] In the opposite figure**

The length of AB = the length of AD

= the length of DC

and $m(\angle ADC) = 80^\circ$

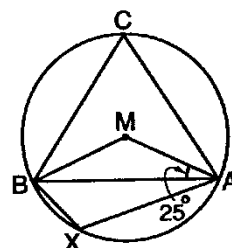
Find : $m(\angle ACB)$ and $m(\angle B)$

**[21] In the opposite figure**

ABC is a triangle inscribed in a circle M

and $m(\angle MAB) = 25^\circ$

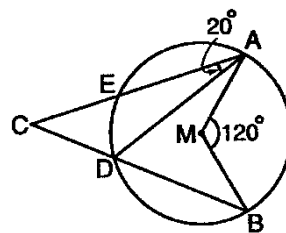
Find : (1) $m(\angle AMB)$ (2) $m(\angle ACB)$
(3) $m(\angle AXB)$ (4) $m(\angle A)$



[22] In the opposite figure

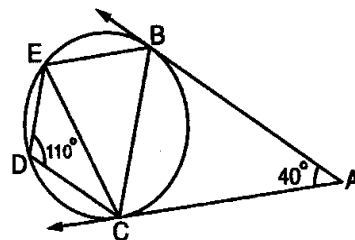
$\overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}$, $m(\angle AMB) = 120^\circ$
and $m(\angle DAC) = 20^\circ$

Find : $m(\angle C)$

**[23] In the opposite figure**

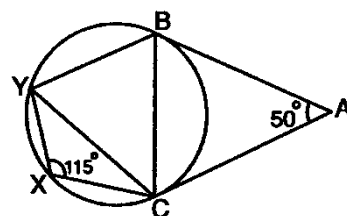
\overrightarrow{AB} and \overrightarrow{AC} are two tangents to a circle at B and C, $m(\angle BAC) = 40^\circ$
and $m(\angle CDE) = 110^\circ$

Prove that : (i) $CB = CE$
(ii) $\overline{BE} \parallel \overline{AC}$

**[24] In the opposite figure**

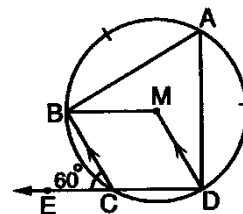
\overline{AB} and \overline{AC} are two tangents to a circle at B and C, $m(\angle A) = 50^\circ$ and $m(\angle CXY) = 115^\circ$

Prove that : (i) \overline{BC} bisects $\angle ABY$
(ii) $CB = CY$

**[25] In the opposite figure**

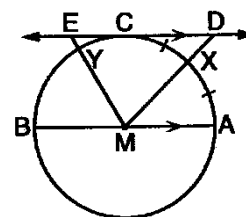
$m(\angle BCE) = 60^\circ$, $\overline{BC} \parallel \overline{MD}$
and A is the midpoint of BD the major

Prove that : (i) BMDC is a rhombus.
(ii) \overline{AC} is a diameter of the circle.

**[26] In the opposite figure**

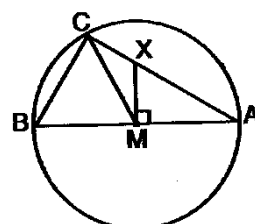
\overline{AB} is a diameter in the circle M.
 \overrightarrow{DE} is a tangent to it at C, $\overline{AB} \parallel \overrightarrow{DE}$
, X is the midpoint of AC and
 $m(\angle BY) = 2m(\angle CY)$

Find : The measures of the angles of the triangle MDE

**[27] In the opposite figure**

\overline{AB} is a diameter in the circle M.
 $\overline{MX} \perp \overline{AB}$

Prove that : $m(\angle AXM) = \frac{1}{2}m(\angle AMC)$

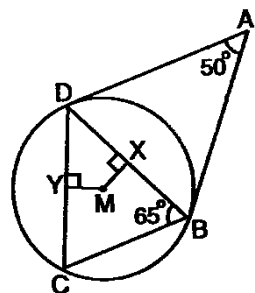


[28] In the opposite figure

\overline{AB} and \overline{AC} are two tangents to a circle M at B and D, $\overline{MX} \perp \overline{BD}$, $\overline{MY} \perp \overline{CD}$

$$m(\angle A) = 50^\circ, m(\angle DBC) = 65^\circ$$

Prove that : $MX = MY$

**[29] In the opposite figure**

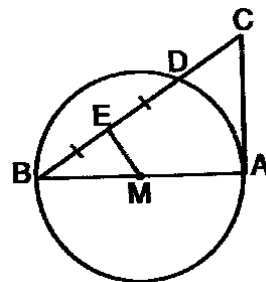
\overline{AB} is a diameter of the circle M

\overline{AC} is a tangent to it at A

and E is the midpoint of \overline{BD}

Prove that : (i) ACEM is a cyclic quad.

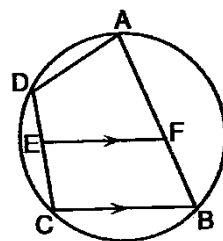
(ii) $\overline{ME} \parallel \overline{AD}$

**[30] In the opposite figure**

ABCD is a cyclic quad.

and $\overline{FE} \parallel \overline{BC}$

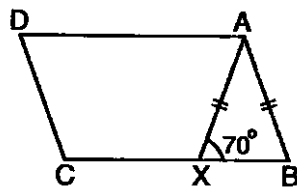
Prove that : AFED is a cyclic quad.

**[31] In the opposite figure**

ABCD is a parallelogram, $AB = AX$

and $m(\angle AXB) = 70^\circ$

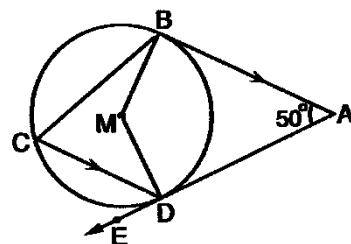
Prove that : AXCD is a cyclic quad.

**[32] In the opposite figure**

\overline{AB} and \overline{AC} are two tangents to a circle M at B and D, $m(\angle A) = 50^\circ$ and $\overline{AB} \parallel \overline{DC}$

(i) **Prove that :** ABMD is a cyclic quad.

(ii) **Find :** $m(\angle ABC)$

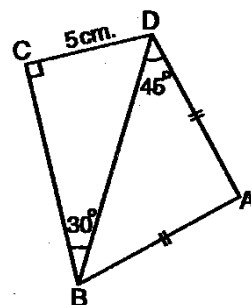
**[33] In the opposite figure**

$AB = AD$, $m(\angle ADB) = 45^\circ$, $m(\angle C) = 90^\circ$

, $m(\angle CBD) = 30^\circ$ and $DC = 5 \text{ cm}$.

(i) **Prove that :** ABCD is a cyclic quad.

(ii) **Calculate** the radius length of the circle passing through the vertices of the figure ABCD



[34] In the opposite figure

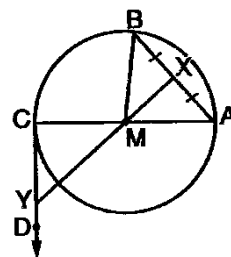
\overline{AB} is a diameter of the circle M

\overrightarrow{CD} is a tangent to it

and X is the midpoint of \overline{AB}

Prove that : (i) AXCY is a cyclic quad.

(ii) $m(\angle BMC) = 2 m(\angle MYC)$

**[35] In the opposite figure**

\overline{AB} is a diameter of the circle M

\overrightarrow{BD} is a tangent to it at B

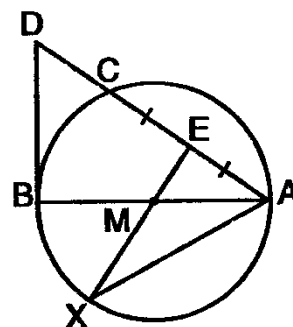
and E is the midpoint of \overline{AC}

Prove that :

(i) MEDB is a cyclic quad.

(ii) $m(\angle BAX) = \frac{1}{2} m(\angle D)$

(iii) \overrightarrow{AB} is a tangent to the circle passing through the points B, C and D

**[36] In the opposite figure**

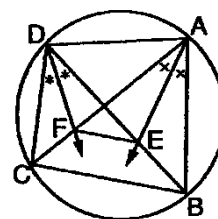
ABCD is a cyclic quad.

\overrightarrow{AE} bisects $\angle BAC$

and \overrightarrow{DF} bisects $\angle BDC$

Prove that : (i) AEFD is a cyclic quad.

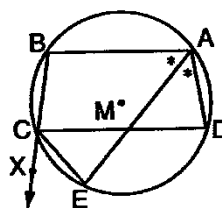
(ii) $\overline{EF} \parallel \overline{BC}$

**[37] In the opposite figure**

ABCD is a cyclic quad.

and \overrightarrow{AE} bisects $\angle A$

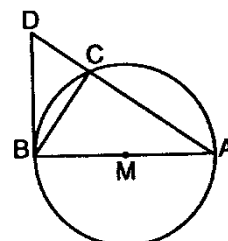
Prove that : \overrightarrow{CE} bisects $\angle XCD$

**[38] In the opposite figure**

\overline{AB} is a diameter of the circle M

\overrightarrow{BD} is a tangent to it at B

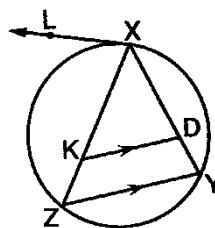
Prove that : \overrightarrow{AB} is a tangent to the circle passing through the vertices of $\triangle BCD$



[39] In the opposite figure

\overleftrightarrow{XY} is a tangent to the circle at X $\overleftrightarrow{DK} \parallel \overleftrightarrow{YZ}$

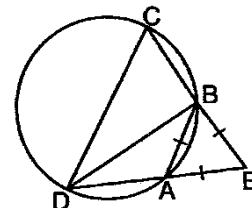
Prove that : \overleftrightarrow{XL} is a tangent to the circle passing through the vertices of $\triangle XDK$

**[40] In the opposite figure**

ABCD is a cyclic quad.

, $BE = EA = AB$ and $\angle B = 60^\circ$

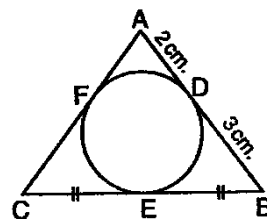
Prove that : \overline{CD} is a diameter of the circle

**[41] In the opposite figure**

$AD = 2\text{cm}$, $DB = 3\text{cm}$.

and E is the mid point of \overline{BC}

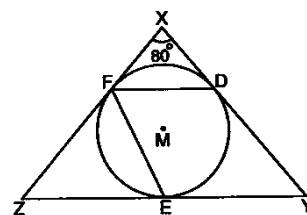
Calculate : The perimeter of $\triangle ABC$

**[42] In the opposite figure**

XYZ is a triangle in which : $m(\angle YXZ) = 80^\circ$

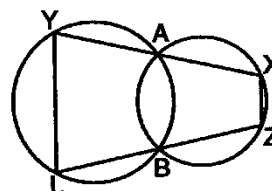
, The circle M touches its sides \overline{XY} , \overline{YZ} and \overline{ZX} at the points D, E and F respectively. If $XY = XZ$

Find : $m(\angle DFE)$

**[43] In the opposite figure**

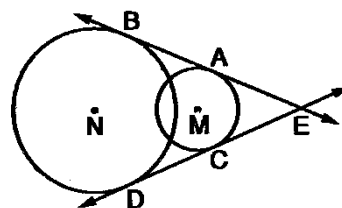
Two circles intersect at A and B

Prove that : $\overline{XZ} \parallel \overline{YL}$

**[44] In the opposite figure**

\overleftrightarrow{AB} and \overleftrightarrow{CD} are two tangents to the two circles M and N, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

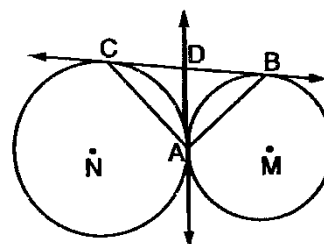
Prove that : $AB = CD$

**[45] In the opposite figure**

M and N are two touching externally circles at A, \overleftrightarrow{BC} is a common tangent to the two circles at B and C

and \overleftrightarrow{AD} is a common tangent to them at A

Prove that : $m(\angle BAC) = 90^\circ$



Model answer

[1] $\because \overline{AB}$ is a diameter

$$\therefore m(\angle ADB) = 90^\circ$$

[inscribed angle drawn in a semi circle]

In $\triangle ABD$

$$m(\angle ABD) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$\because \overrightarrow{BD}$ bisects $\angle ABC$

$$\therefore m(\angle CBD) = m(\angle DBA) = 30^\circ$$

$\because ABCD$ is a cyclic quad.

$$\therefore m(\angle A) + m(\angle C) = 180^\circ$$

[two opposite angles]

$$\therefore m(\angle C) = 180^\circ - 60^\circ = 120^\circ$$

In $\triangle BCD$

$$m(\angle CDB) = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$$

[2]

$$\because AC = BD$$

$$\therefore AC = BD$$

by subtracting $m(\angle C)$ from both terms

$$\therefore m(\angle A) = m(\angle C)$$

$$\therefore AB = CD$$

[3]

$\because BCDE$ is a cyclic quad.

$$\therefore m(\angle C) + m(\angle BED) = 180^\circ$$

[two opposite angles]

$$\therefore m(\angle C) = 180^\circ - 110^\circ = 70^\circ$$

$\because ABCD$ is a parallelogram

$$\therefore m(\angle A) = m(\angle C) = 70^\circ$$

[two opposite angles]

$$\therefore m(\angle AED) = 180^\circ \quad (\text{st. angle})$$

$$\therefore m(\angle AEB) = 180^\circ - 110^\circ = 70^\circ$$

In $\triangle ABE$

$$\therefore m(\angle BAE) = m(\angle BEA) = 70^\circ$$

$\therefore \triangle ABE$ is an isos. triangle

$$\therefore m(\angle ABE) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$$

$\because ABCD$ is a parallelogram

$$\therefore m(\angle A) + m(\angle D) = 180^\circ$$

[two consecutive angles]

$$\therefore m(\angle D) = 180^\circ - 70^\circ = 110^\circ$$

$$\therefore m(\angle EBC) = 2m(\angle D) = 2 \times 110^\circ = 220^\circ \quad [\text{inscribed angle and opposite arc}]$$

[4]

Const.: Draw \overline{BC}

$\because \overline{BE}$ is a tangent to the circle at B

$$\therefore \overline{EB} \perp \overline{AB}$$

$$\therefore m(\angle EBA) = 90^\circ$$

$\because D$ is a midpoint of \overline{AC}

$$\therefore \overline{MD} \perp \overline{AC}$$

$$\therefore m(\angle MDE) = 90^\circ$$

$$\therefore m(\angle EBA) + m(\angle MDE) = 90^\circ + 90^\circ = 180^\circ$$

$\therefore MBED$ is a cyclic quad.

$$\therefore m(\angle ACB) = 90^\circ$$

[inscribed angle drawn in a semi circle]

$$\therefore m(\angle ACB) = m(\angle ADM) = 90^\circ$$

[and they are in position of corresponding angles]

$$\therefore \overline{MD} \parallel \overline{BC}$$

[5]

$\therefore \overrightarrow{FA}$ is a tangent to the circle

$$\therefore m(\angle FAB) = m(\angle ACB) \rightarrow (1)$$

[angle of tangency and inscribed angle subtended by AB]

$$\therefore \overline{DE} \parallel \overline{FA}$$

$$\therefore m(\angle FAB) = m(\angle ADE) \rightarrow (2)$$

[alt. angles]

From (1) and (2)

$$\therefore m(\angle ACB) = m(\angle ADE)$$

[Exterior angle and interior angle at opposite vertex]

$\therefore DBCE$ is a cyclic quad.

[6]

$\therefore \overrightarrow{XA}$ and \overrightarrow{XB} are two tangents drawn from X

$$\therefore XA = XB$$

In $\triangle XAB$

$$\therefore XA = XB \text{ (proved)}$$

$$\therefore m(\angle XAB) = m(\angle XBA) = \frac{180^\circ - 70^\circ}{2} = 55^\circ \rightarrow (1)$$

$\therefore ABCD$ is a cyclic quad.

$$\therefore m(\angle A) + m(\angle C) = 180^\circ$$

[two opposite angles]

$$\therefore m(\angle BAD) = 180^\circ - 125^\circ = 55^\circ$$

$\rightarrow (2)$

From (1) and (2)

$$\therefore m(\angle XAB) = m(\angle BAD) = 55^\circ$$

$\therefore \overrightarrow{AB}$ bisects $\angle DAX$

$$\therefore m(\angle DAX) + m(\angle X) = 110^\circ + 70^\circ = 180^\circ$$

[and they are in position of interior supp. angles]

$$\therefore \overline{AD} \parallel \overline{XB}$$

[7]

$$\therefore m(\angle CBD) + m(\angle CAD) = 20^\circ$$

[two inscribed angles subtended by the same arc CD]

\therefore In $\triangle ADE$

$$m(\angle ADE) = 180^\circ - [15^\circ + 20^\circ] = 145^\circ$$

$$\therefore m(\angle BDE) = 180^\circ \text{ (st. angle)}$$

$$\therefore m(\angle ADB) = 180^\circ - 145^\circ = 35^\circ$$

$\therefore \angle AFB$ is an exterior angle to $\triangle BDF$

$$\therefore m(\angle AFB) = m(\angle FDB) + m(\angle FBD) = 35^\circ + 20^\circ = 55^\circ$$

[8]

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore m(\angle ADB) = m(\angle DBC) = 40^\circ$$

[alt. angles]

$\therefore \angle AFB$ is an exterior angle to $\triangle AFD$

$$\therefore m(\angle DAC) = 80^\circ + 40^\circ = 120^\circ$$

$$\therefore m(\angle DAC) + m(\angle DBC) = 120^\circ + 40^\circ = 160^\circ \text{ [and they are drawn on the same base } \overline{CD} \text{ and in the same side of it]}$$

$\therefore ABCD$ is a cyclic quad.

[9]

$\therefore \overline{AC}$ is a diameter and \overrightarrow{CY} is a tangent to the circle

$$\therefore \overrightarrow{CY} \perp \overline{AC}$$

$$\therefore m(\angle ACY) = 90^\circ$$

$\therefore X$ is a midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

$$\therefore m(\angle AXY) = 90^\circ$$

$\therefore m(\angle ACY) = m(\angle AXY) = 90^\circ$ [and they drawn on the same base \overline{AY} and in the same side of it]

$\therefore AXCY$ is a cyclic quad.

$$\therefore m(\angle MYC) = m(\angle BAC) \rightarrow (1) \quad [\text{drawn on the same base } \overline{XC} \text{ and in the same side of it}]$$

$$\therefore m(\angle BMC) = 2m(\angle BAC) \rightarrow (2) \quad [\text{Central and inscribed angles sub. by } BC]$$

From (1) and (2)

$$\therefore m(\angle BMC) = 2m(\angle MYC)$$

[10]

$\therefore ABCD$ is a parallelogram

$$\therefore m(\angle B) = m(\angle D) \rightarrow (1)$$

In $\triangle ABE$

$$\therefore AB = AE$$

$$\therefore m(\angle B) = m(\angle AEB) \rightarrow (2)$$

From (1) and (2)

$$\therefore m(\angle AEB) = m(\angle D) \quad [\text{Exterior angle and interior angle at opposite vertex}]$$

$\therefore AECD$ is a cyclic quad.

$\therefore ABCD$ is a parallelogram

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore m(\angle DAE) = m(\angle AEB) \rightarrow (3) \quad [\text{alt. angles}]$$

From (2) and (3)

$$\therefore m(\angle DAE) = m(\angle B)$$

$\therefore \overline{AD}$ is a tangent to the circumcircle of $\triangle ABE$

[11]

$$\therefore \overline{MC} \perp \overline{AB}$$

$$\therefore m(\angle DMB) = 90^\circ \rightarrow (1)$$

$\therefore \overline{AB}$ is a diameter

$$\therefore m(\angle AEB) = 90^\circ \quad [\text{inscribed angle drawn in a semi circle}]$$

$$\therefore m(\angle DEB) = 180^\circ - 90^\circ = 90^\circ \rightarrow (2) \quad [\text{st. angle}]$$

From (1) and (2)

$$\therefore m(\angle DMB) = m(\angle AEB) = 90^\circ \quad [\text{and they drawn on the same base } \overline{BD} \text{ and in the same side of it}]$$

$\therefore DEMB$ is a cyclic quad.

$$\therefore m(\angle EBM) = m(\angle EDM) \rightarrow (3) \quad [\text{drawn on the same base } \overline{ME} \text{ and in the same side of it}]$$

In $\triangle MEB$

$$\therefore MB = ME \quad (2 \text{ radii})$$

$$\therefore m(\angle EBM) = m(\angle MEB) \rightarrow (4)$$

From (3) and (4)

$$\therefore m(\angle MEB) = m(\angle EDM)$$

$\therefore \overline{EM}$ is a tangent to the circumcircle of $\triangle NDE$

[12]

Const.: Draw \overline{BD}

$\therefore \overline{BD}$ is a diameter

$$\therefore m(\angle ADB) = 90^\circ$$

$\therefore ABCD$ is a cyclic quad.

$$\therefore m(\angle BCD) + m(\angle A) = 180^\circ \quad [\text{two opposite angles}]$$

$$\therefore m(\angle BCD) = 180^\circ - 50^\circ = 130^\circ$$

$$\therefore m(\widehat{BC}) = m(\widehat{CD})$$

$$\therefore BC = CD$$

In $\triangle BCD$

$$\therefore BC = CD(\text{proved})$$

$$\therefore m(\angle CDB) = m(\angle CBD) = \frac{180^\circ - 130^\circ}{2} = 25^\circ \rightarrow (2)$$

From (1) and (2)

$$\therefore m(\angle CDA) = 90^\circ + 25^\circ = 115^\circ$$

[13]

$$\therefore m(\angle ABE) = 30^\circ$$

$$\therefore m(\widehat{AE}) = 2 \times 30^\circ = 60^\circ \quad [\text{opposite arc}]$$

$\therefore \overline{BD}$ is a diameter

$$\therefore m(\angle AEB) = 180^\circ \quad [\text{semi circle}]$$

$$\therefore m(\angle EDB) = 180^\circ - 60^\circ = 120^\circ$$

$\therefore \overline{CD}$ is a tangent and $\overline{CD} \parallel \overline{BE}$

$$\therefore m(\widehat{BD}) = m(\widehat{DE}) = \frac{1}{2}m(\widehat{BE}) = 120^\circ \div 2 = 60^\circ$$

[14]

$$\therefore m(\angle BCD) = 35^\circ$$

$$\therefore m(\widehat{BD}) = 2 \times 35^\circ = 70^\circ \quad [\text{opposite arc}]$$

$\therefore \overline{XY}$ is a tangent and $\overline{AB} \parallel \overline{XY}$

$$\therefore m(\widehat{AD}) = m(\widehat{BD}) = 70^\circ$$

$$\therefore m(\angle ADB) = 2 \times 70^\circ = 140^\circ$$

$\therefore \angle AMB$ is a central angle subtended by ADB

$$\therefore m(\angle AMB) = 140^\circ$$

In $\triangle AMB$

$$\therefore AM = BM \quad (2 \text{ radii})$$

$$\therefore m(\angle ABM) = m(\angle BAM) = \frac{180^\circ - 140^\circ}{2} = 20^\circ$$

[15]

$\therefore \overline{XZ}$ and \overline{XY} are two tangents drawn from X

$$\therefore XZ = XY$$

In $\triangle XYZ$

$$\therefore XY = XZ \quad (\text{proved})$$

$$\therefore m(\angle XZY) = m(\angle XYZ) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$$

$$\therefore m(\angle ZEY) = m(\angle XYZ) = 50^\circ \rightarrow (1) \quad \left[\text{angle of tangency and inscribed angle subtended by } ZY \right]$$

$\therefore EDZY$ is a cyclic quad.

$$\therefore m(\angle Y) + m(\angle D) = 180^\circ \quad [\text{two opposite angles}]$$

$$\therefore m(\angle EYZ) = 180^\circ - 130^\circ = 50^\circ \rightarrow (2)$$

From (1) and (2)

[16]

$\therefore \overline{AZ}, \overline{AX}$ are two tangent – segment to the circle

$$\therefore AZ = AX = 2 \text{ cm.}$$

$\therefore \overline{BY}, \overline{BX}$ are two tangent – segment to the circle

$$\therefore BY = BX = 5 \text{ cm.}$$

$\therefore \overline{CY}, \overline{CZ}$ are two tangent – segment to the circle

$$\therefore CY = CZ = 3 \text{ cm.}$$

$$\therefore \text{The per. of } \triangle ABC = 2 + 3 + 3 + 5 + 5 + 2 = 20 \text{ cm.}$$

Const : Draw \overline{MX} and \overline{MY}

$\therefore \overline{MX}$ is a radius, \overline{BX} is a tangent – segment to the circle at X

$$\therefore \overline{MX} \perp \overline{BX}$$

$$\therefore m(\angle BXM) = 90^\circ$$

similarly $\therefore m(\angle BYM) = 90^\circ$

$$\therefore m(\angle BXM) + m(\angle BYM) = 90^\circ + 90^\circ = 180^\circ$$

$\therefore BXYM$ is a cyclic quad.

$$\therefore m(\angle XMY) + m(\angle XBY) = 180^\circ \quad [\text{two opp. angles}]$$

$$\therefore m(\angle XMY) = 180^\circ - 40^\circ = 140^\circ$$

$$\therefore m(\angle XZY) = \frac{1}{2}m(\angle XMY) = \frac{1}{2} \times 140^\circ = 70^\circ$$

[inscribed angle and central angle subtended by xy]

In $\triangle EYZ$

$$\therefore m(\angle EYZ) + m(\angle ZEY)$$

$$\therefore ZE = ZY$$

$$\therefore m(\angle ZXY) + m(\angle XYE) = 80 + 50 + 50 = 180^\circ \quad [\text{and they are in position of interior supp. angles}]$$

$$\therefore \overline{YE} \parallel \overline{XZ}$$

[17]

\therefore ABCD is a rectangle

$$\therefore AB = DC \quad (\text{two opposite sides})$$

$$\therefore DE = DC \quad (\text{given})$$

$$\therefore DE = AB$$

$$\therefore m(\overline{DE}) = m(\overline{AB})$$

By adding $m(\overline{AE})$ for both terms

$$\therefore m(\overline{AD}) = m(\overline{BE})$$

$$\therefore AD = BE$$

[18]

$$\therefore m(\angle BAC) = m(\angle BEC) \quad \left[\text{two inscribed angles sub. by the same arc } BC \right]$$

$$\therefore m(\angle BAC) = 25^\circ$$

$\therefore \overline{AB}$ is a diameter

$$\therefore m(\angle ACB) = 90^\circ \quad [\text{inscribed angle drawn in a semi circle}]$$

In $\triangle ABC$

$$m(\angle ABC) = 180^\circ - (90^\circ + 25^\circ) = 65^\circ$$

\therefore ABCD is a cyclic quad.

$$\therefore m(\angle ABC) + m(\angle ADC) = 180^\circ \quad [\text{two opposite angles}]$$

$$\therefore m(\angle ADC) = 180^\circ - 65^\circ = 115^\circ$$

[19]

\therefore XYZL is a cyclic quad.

$$\therefore m(\angle L) + m(\angle Y) = 180^\circ \quad [\text{two opposite angles}]$$

$$\therefore m(\angle L) = 180^\circ - 60^\circ = 120^\circ$$

$\therefore \overline{XY}$ is a diameter

$$\therefore m(\angle XZY) = 90^\circ$$

In $\triangle XYZ$

$$m(\angle ZXY) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

In $\triangle LXZ$

$$\therefore LX = LZ$$

$$\therefore m(\angle LXZ) = m(\angle LZX) = \frac{180^\circ - 120^\circ}{2} = 30^\circ$$

[20]

$$\therefore \text{length of } AB = \text{length of } AD = \text{length of } CD$$

$$\therefore AB = AD = CD$$

In $\triangle ACD$

$$\therefore AD = CD$$

$$\therefore m(\angle ACD) = m(\angle CAD) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$$

$$\therefore m(\overline{AB}) = m(\overline{AD}) = m(\overline{CD}) = 2 \times 50^\circ = 100^\circ \quad [\text{arc and opp. inscribed angles}]$$

$$\therefore m(\angle ACB) = \frac{1}{2}m(\widehat{AB}) = 50^\circ \quad [\text{inscribed angle and opp. arc}]$$

$$\therefore m(\widehat{BC}) = 360^\circ - (100^\circ + 100^\circ + 100^\circ) = 60^\circ$$

[21]

In ΔAMB

$$\therefore AM = BM \quad [2 \text{ radii}]$$

$$\therefore m(\angle MAB) = m(\angle MBA) = 25^\circ$$

$$\therefore m(\angle AMB) = 180 - (25^\circ + 25^\circ) = 130^\circ$$

$$\therefore m(\angle ACB) = \frac{1}{2}m(\angle AMB) \quad \left[\text{inscribed angle and central angle subtended by } \widehat{AB} \right]$$

$$\therefore m(\angle ACB) = \frac{1}{2} \times 130 = 65^\circ$$

 \therefore AXBC is a cyclic quad.

$$\therefore m(\angle AXB) + m(\angle ACB) = 180^\circ \quad [\text{two opposite angles}]$$

$$\therefore m(\angle ADC) = 180^\circ - 65^\circ = 115^\circ$$

$$\therefore m(\widehat{AB}) = m(\angle AMB) \quad [\text{opposite central angle}]$$

$$\therefore m(\widehat{AB}) = 130^\circ$$

[22]

$$\therefore m(\angle ADB) = \frac{1}{2}m(\angle AMB) \quad \left[\text{inscribed angle and central angle subtended by } \widehat{AB} \right]$$

$$\therefore m(\angle ADB) = \frac{1}{2} \times 120 = 60^\circ$$

 $\therefore \angle ADB$ is an exterior angle to ΔACD

$$\therefore m(\angle C) = m(\angle ADB) - m(\angle DAC)$$

$$\therefore m(\angle C) = 60^\circ - 20^\circ = 40^\circ$$

[23]

In ΔABC

$$\therefore AB = AC \quad [\text{two tangent – segments drawn from A}]$$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$$

 $\therefore \overrightarrow{AC}$ is a tangent.

$$\therefore m(\angle ACB) = m(\angle BEC) = 70^\circ \quad [\text{inscribed angle and angle of tangency subtended by } \widehat{BC}]$$

 \therefore BCDE is a cyclic quad.

$$\therefore m(\angle CBE) + m(\angle CDE) = 180^\circ \quad [\text{two opposite angles}]$$

$$\therefore m(\angle CBE) = 180^\circ - 110^\circ = 70^\circ$$

In ΔCBE

$$\therefore m(\angle CBE) = m(\angle CEB) = 70^\circ$$

$$\therefore CB = CE \quad [\text{isos. triangle}]$$

$$\therefore m(\angle ACB) = m(\angle CBE) = 70^\circ \quad [\text{and they are in position of alt. angles}]$$

$$\therefore \overrightarrow{BE} \parallel \overrightarrow{AC}$$

[24]

In ΔABC

$$\therefore AB = AC \quad [\text{two tangent – segments drawn from A}]$$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

 $\therefore \overrightarrow{AC}$ is a tangent.

$$\therefore m(\angle ACB) = m(\angle CYB) = 65^\circ \quad [\text{inscribed angle and angle of tangency subtended by } \widehat{BC}]$$

- \therefore BCXY is a cyclic quad.
 $\therefore m(\angle CXY) + m(\angle CBY) = 180^\circ$ [two opposite angles]
 $\therefore m(\angle CBE) = 180^\circ - 115^\circ = 65^\circ$
 $\therefore m(\angle ABC) = m(\angle CBY) = 65^\circ$
 $\therefore \overrightarrow{BC}$ bisects $\angle ABY$
 In ΔCBY
 $\therefore m(\angle CBY) = m(\angle CYB) = 65^\circ$
 $\therefore CB = CY$ [isos. triangle]
 [25]
 $\therefore \overrightarrow{BC} \parallel \overrightarrow{MD} \rightarrow (1)$
 $\therefore m(\angle BCE) = m(\angle MDC)$ [corresponding. angles]
 \therefore ABCD is a cyclic quad.
 $\therefore m(\angle BCY) = m(\angle DAB) = 60^\circ$ [an exterior angle at opp. vertex]
 $\therefore m(\angle BMD) = 2m(\angle DAB)$ [central angle and inscribed angle subtended by BD]
 $\therefore m(\angle ACB) = 2 \times 60^\circ = 120^\circ$
 $\therefore m(\angle BMD) + m(\angle MDC) = 120^\circ + 60^\circ = 180^\circ$ [and they are in position of interior supp. angles]
 $\therefore m(\angle ADC) = 180^\circ - 65^\circ = 115^\circ$
 $\therefore \overrightarrow{MB} \parallel \overrightarrow{DC} \rightarrow (2)$
 From (1) and (2)
 \therefore BMDC is a parallelogram
 $\therefore MD = MB$ [two radii]
 \therefore BMDC is a rhombus
 $\therefore m(\angle A) = m(\angle C) = 70^\circ$
 $\therefore m(\angle BCD) = m(\angle BMD) = 120^\circ$ [central angle and opp. arc]
 $\therefore BC = CD$ [two adjacent sides of rhombus]
 $\therefore m(\angle BC) = m(\angle CD) = \frac{1}{2} \times 120^\circ = 60^\circ$
 $\therefore m(\angle BAD) = 360^\circ - 120^\circ = 240^\circ$
 \therefore A is a mid point of BD
 $\therefore m(\angle BA) = m(\angle AD) = \frac{1}{2} \times 240^\circ = 120^\circ$
 $\therefore m(\angle AC) = m(\angle BC) + m(\angle BA) = 60^\circ + 120^\circ = 180^\circ$
 $\therefore \overrightarrow{AC}$ is a diameter of the circle
 [26]
 $\therefore \overrightarrow{AB}$ is a diameter
 $\therefore m(\angle ACB) = 180^\circ$ [semi circle]
 $\therefore \overrightarrow{AB} \parallel \overrightarrow{DE}$
 $\therefore m(\angle AC) = m(\angle BC) = \frac{1}{2} \times 180^\circ = 90^\circ$
 \therefore X is a mid point of AC
 $\therefore m(\angle AX) = m(\angle CX) = \frac{1}{2} \times 90^\circ = 45^\circ$
 $\therefore m(\angle BY) = 2m(\angle CY)$

$$\therefore m(\widehat{BY}) = 60^\circ$$

$$\therefore m(\widehat{CY}) = 30^\circ$$

$$\therefore m(\widehat{XY}) = m(\widehat{CX}) + m(\widehat{CY}) = 45^\circ + 30^\circ = 75^\circ$$

$$\therefore m(\angle XMY) = m(\widehat{XY}) = 75^\circ \quad [\text{Central angle and opposite arc}]$$

$$\therefore m(\angle AMX) = \frac{1}{2} m(\widehat{AX}) = \frac{1}{2} \times 45^\circ = 22.5^\circ \quad [\text{inscribed angle and opposite arc}]$$

$$\therefore \overline{AB} \parallel \overline{DE}$$

$$\therefore m(\angle CDM) = m(\angle AMD) = 22.5^\circ \quad [\text{alt. angles}]$$

In $\triangle MDE$

$$\therefore m(\angle DEM) = 180 - (75^\circ + 22.5^\circ) = 82.5^\circ$$

[27]

$\therefore \overline{AB}$ is a diameter

$$\therefore m(\angle ACB) = 90^\circ \quad [\text{inscribed angle drawn in a semi circle}]$$

$$\therefore m(\angle AMX) = m(\angle ACB) = 90^\circ \quad [\text{an exterior angle and interior angle at opp. vertex}]$$

$\therefore BCXM$ is a cyclic quad.

$$\therefore m(\angle AXM) = m(\angle ABC) \rightarrow (1) \quad [\text{an exterior angle at opp. vertex}]$$

$$\therefore m(\angle ABC) = \frac{1}{2} m(\angle AMC) \rightarrow (2) \quad [\text{inscribed angle and central angle subtended by } AC]$$

From (1) and (2)

$$\therefore m(\angle AXM) = \frac{1}{2} m(\angle AMC)$$

[28] In $\triangle ABD$

$$\therefore AB = AD \quad [\text{two tangent – segments drawn from } A]$$

$$\therefore m(\angle ABD) = m(\angle ADB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

$\therefore \overline{AB}$ is a tangent.

$$\therefore m(\angle ABD) = m(\angle BCD) = 65^\circ \quad [\text{inscribed angle and angle of tangency subtended by } BD]$$

In $\triangle DBC$

$$\therefore m(\angle DBC) = m(\angle DCB) = 65^\circ$$

$$\therefore DB = DC \rightarrow (1) \quad [\text{isos. triangle}]$$

$$\therefore \overline{MX} \perp \overline{BD}, \overline{MY} \perp \overline{CD} \rightarrow (2)$$

From (1) and (2)

$$\therefore MX = MY$$

[29] Const.: Draw \overline{AD}

$\therefore \overline{AC}$ is a tangent to the circle at A

$$\therefore \overline{CA} \perp \overline{AB}$$

$$\therefore m(\angle CAM) = 90^\circ$$

$\therefore E$ is a midpoint of \overline{BD}

$$\therefore \overline{ME} \perp \overline{BD}$$

$$\therefore m(\angle MEC) = 90^\circ$$

$$\therefore m(\angle CAM) + m(\angle MEC) = 90^\circ + 90^\circ = 180^\circ$$

$\therefore ACEM$ is a cyclic quad.

$$\therefore m(\angle ADB) = 90^\circ \quad [\text{inscribed angle drawn in a semi circle}]$$

$$\therefore m(\angle ADB) = m(\angle MEB) = 90^\circ \quad [\text{and they are in position of corresponding angles}]$$

$$\therefore \overline{ME} \parallel \overline{AD}$$

[30]

 \therefore ABCD is a cyclic quad.

$$\therefore m(\angle A) + m(\angle C) = 180^\circ \rightarrow (1) \quad [\text{two opposite angles}]$$

$$\therefore \overline{FE} \parallel \overline{BC}$$

$$\therefore m(\angle EFC) + m(\angle C) = 180^\circ \rightarrow (2) \quad [\text{interior supp. angles}]$$

From (1) and (2)

$$\therefore m(\angle A) = m(\angle EFC) \quad [\text{and they are an exterior angle and interior angle at opp. vertex}]$$

 \therefore AFED is a cyclic quad.

[31]

In ΔABX

$$\therefore AB = AX \quad [\text{isos. triangle}]$$

$$\therefore m(\angle ABX) = m(\angle AXB) = 70^\circ$$

 \therefore ABCD is a parallelogram

$$\therefore m(\angle B) + m(\angle D) = 70^\circ \quad [\text{two opposite angles}]$$

$$\therefore m(\angle AXB) = m(\angle D) = 70^\circ \quad [\text{and they are an exterior angle and interior angle at opp. vertex}]$$

 \therefore AXCD is a cyclic quad.

[32]

 $\therefore \overline{MB}$ is a radius, \overline{AB} is a tangent – segment to the circle at B

$$\therefore \overline{MB} \perp \overline{AB}$$

$$\therefore m(\angle ABM) = 90^\circ$$

$$\text{similarly } \therefore m(\angle ADM) = 90^\circ$$

$$\therefore m(\angle ABM) + m(\angle ADM) = 90^\circ + 90^\circ = 180^\circ$$

 \therefore ABMD is a cyclic quad.

$$\therefore m(\angle BAD) + m(\angle BMD) = 180^\circ \quad [\text{two opp. angles}]$$

$$\therefore m(\angle BMD) = 180^\circ - 50^\circ = 130^\circ$$

$$\therefore m(\angle BCD) = \frac{1}{2}m(\angle BMD) = \frac{1}{2} \times 130^\circ = 65^\circ \quad [\text{inscribed angle and central angle subtended by } BD]$$

$$\therefore \overline{AB} \parallel \overline{DC}$$

$$\therefore m(\angle ABC) + m(\angle BCD) = 180^\circ \quad [\text{interior supp. angles}]$$

$$\therefore m(\angle ABC) = 180^\circ - 65^\circ = 115^\circ$$

[33]

In ΔABD

$$\therefore AD = AB$$

$$\therefore m(\angle ADB) = m(\angle ABD) = 45^\circ$$

$$\therefore m(\angle BAD) = 180^\circ - (45^\circ + 45^\circ) = 90^\circ$$

$$\therefore m(\angle BAD) + m(\angle BCD) = 90^\circ + 90^\circ = 180^\circ$$

 \therefore ABCD is a cyclic quad. $\therefore \overline{BD}$ is the diameter to the circle which passes through ABCDIn ΔABD

$$\therefore m(\angle BCD) = 90^\circ \text{ and } m(\angle CBD) = 30^\circ$$

$$\therefore BD = 2CD = 2 \times 5 = 10 \text{ cm.}$$

[34]

 $\therefore \overline{AC}$ is a diameter and \overline{CY} is a tangent to the circle

$$\therefore \overline{CY} \perp \overline{AC}$$

$$\therefore m(\angle ACY) = 90^\circ$$

 \therefore X is a midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

$$\therefore m(\angle AXY) = 90^\circ$$

- $\therefore m(\angle ACY) = m(\angle AXY) = 90^\circ$ [and they drawn on the same base \overline{AY} and in the same side of it]
 $\therefore AXCY$ is a cyclic quad.
 $\therefore m(\angle MYC) = m(\angle BAC) \rightarrow (1)$ [drawn on the same base \overline{XC} and in the same side of it]
 $\therefore m(\angle BMC) = 2m(\angle BAC) \rightarrow (2)$ [Central and inscribed angles sub. by BC]

From (1) and (2)

$$\therefore m(\angle BMC) = 2m(\angle MYC)$$

[35]

- $\therefore \overline{AB}$ is a diameter and \overline{BD} is a tangent to the circle
 $\therefore \overline{BD} \perp \overline{AB}$
 $\therefore m(\angle ABD) = 90^\circ$
 $\therefore E$ is a midpoint of \overline{AC}
 $\therefore \overline{ME} \perp \overline{AC}$
 $\therefore m(\angle MED) = 90^\circ$
 $\therefore m(\angle MBD) + m(\angle MED) = 90^\circ + 90^\circ = 180^\circ$ [and they are two opp. angles]
 $\therefore MEDB$ is a cyclic quad.
 $\therefore m(\angle BMX) = m(\angle D) \rightarrow (1)$ [an exterior and interior angle at opp. vertex]
 $\therefore m(\angle BAX) = \frac{1}{2}m(\angle BMX) \rightarrow (2)$ [Central and inscribed angles sub. by BX]

From (1) and (2)

$$\therefore m(\angle BAX) = \frac{1}{2}m(\angle D)$$

$\therefore MEDB$ is a cyclic quad.

$$\therefore m(\angle AME) = m(\angle D) \rightarrow (3) \quad \text{[an exterior and interior angle at opp. vertex]}$$

In $\triangle ABC$

- $\therefore E$ and M are two midpoints of \overline{AC} and \overline{AB}
 $\therefore \overline{ME} \parallel \overline{BC}$
 $\therefore m(\angle AME) = m(\angle ABC) \rightarrow (4)$ [corres. angles]

From (3) and (4)

$$\therefore m(\angle ABC) = m(\angle D)$$

$\therefore \overline{AB}$ is a tangent to the circle passing through the points B, C and D

[36]

- $\therefore ABCD$ is a cyclic quad.
 $\therefore m(\angle BAC) = m(\angle BDC)$ (drawn on the same base \overline{BC}) $\rightarrow (1)$
 $\therefore \overline{AE}$ bisects $\angle BAC$
 $\therefore m(\angle EAF) = \frac{1}{2}m(\angle BAC) \rightarrow (2)$
 $\therefore \overline{DF}$ bisects $\angle BDC$
 $\therefore m(\angle EDF) = \frac{1}{2}m(\angle BDC) \rightarrow (3)$

From (1), (2) and (3)

$$\therefore m(\angle EAF) = m(\angle EDF) \quad \text{(and they are drawn on } \overline{EF} \text{ and on one side of it)}$$

$\therefore AEFD$ is a cyclic quad.

$\therefore ABCD$ is a cyclic quad.

$$\therefore m(\angle DAC) = m(\angle DBC) \text{ (drawn on the same base } \overline{DC}) \rightarrow (4)$$

$\therefore AEFD$ is a cyclic quad.

$$\therefore m(\angle DAC) = m(\angle DEF) \text{ (drawn on the same base } \overline{DF}) \rightarrow (5)$$

From (4) and (5)

$\therefore m(\angle DEF) = m(\angle DBC)$ (and they are in position of coresponding angles)

$\therefore \overline{EF} \parallel \overline{BC}$

[37]

\therefore AECB is a cyclic quad.

$\therefore m(\angle ECX) = m(\angle EAB) \rightarrow (1)$ [an exterior and interior angle at opp. vertex]

$\therefore m(\angle DAE) = m(\angle DCE) \rightarrow (2)$ [two inscribed angles sub. by DE]

$\therefore \overline{EA}$ bisects $\angle DAB$

$\therefore m(\angle DAE) = m(\angle EAB) \rightarrow (3)$

From (1), (2) and (3)

$\therefore m(\angle DCE) = m(\angle ECX)$

$\therefore \overline{CE}$ bisects $\angle XCD$

[38]

$\therefore \overline{AB}$ is a diameter

$\therefore m(\angle ACB) = 90^\circ$

$\therefore \overline{AB}$ is a diameter and \overline{BD} is a tangent to the circle

$\therefore \overline{BD} \perp \overline{AB}$

$\therefore m(\angle ABD) = 90^\circ$

$\therefore \overline{BD}$ is a tangent.

$\therefore m(\angle BAD) = m(\angle DBC)$ [inscribed angle and angle of tangency subtended by BC]

In $\triangle ABC$ and $\triangle DBC$

$\therefore m(\angle ACB) = m(\angle ABD) = 90^\circ$

$\therefore m(\angle BAD) = m(\angle DBC)$ (*proved*)

$\therefore m(\angle ABC) = m(\angle BDC)$

$\therefore \overline{AB}$ is a tangent to the circle passing through the vertices of $\triangle BCD$

[39]

$\therefore \overline{LX}$ is a tangent to the circle

$\therefore m(\angle LXZ) = m(\angle XYZ) \rightarrow (1)$ [inscribed angle and angle of tangency subtended by AB]

$\therefore \overline{KD} \parallel \overline{EC}$

$\therefore m(\angle XDK) = m(\angle XYZ) \rightarrow (2)$ [two corresponding angles]

From (1) and (2)

$\therefore m(\angle LXZ) = m(\angle XDK)$

$\therefore \overline{XL}$ is a tangent to the circle passing through the vertices of $\triangle XDK$

[40]

In $\triangle ABE$

$\therefore AE = EB = AB$

$\therefore m(\angle EAB) = m(\angle ABE) = m(\angle AEB) = 60^\circ$

\therefore ABCD is a cyclic quad.

$\therefore m(\angle EAB) = m(\angle C) = 60^\circ$ [an exterior and interior angle at opp. vertex]

$\therefore m(\angle BDC) = \frac{1}{2}m(\angle BAC) = 30^\circ$ [inscribed angle and opp. arc]

\therefore In $\triangle BCD$

$m(\angle DBC) = 180^\circ - [60^\circ + 30^\circ] = 90^\circ$

$\therefore \overline{CD}$ is a diameter

[41]

$\therefore \overline{AD}, \overline{AF}$ are two tangent – segment to the circle

- $\therefore AD = AF = 2 \text{ cm.}$
 $\therefore \overline{BD}, \overline{BE}$ are two tangent – segment to the circle
 $\therefore BD = BE = 3 \text{ cm.}$
 $\therefore E$ is a midpoint of \overline{BC}
 $\therefore BE = EC = 3 \text{ cm.}$
 $\therefore \overline{CE}, \overline{CF}$ are two tangent – segment to the circle
 $\therefore CE = CF = 3 \text{ cm.}$
 \therefore The per. of $\triangle ABC = 2 + 3 + 3 + 3 + 3 + 2 = 16 \text{ cm.}$

[42]

In $\triangle XDF$

- $\therefore XD = XF$ [two tangent – segments drawn from X]

$$\therefore m(\angle XDF) = m(\angle XFD) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$$

In $\triangle XYZ$

- $\therefore XY = XZ$

$$\therefore m(\angle XYZ) = m(\angle XZY) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$$

In $\triangle ZFE$

- $\therefore ZF = ZE$ [two tangent – segments drawn from X]

$$\therefore m(\angle ZFE) = m(\angle ZEF) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

$$\therefore m(\angle XFZ) = 180^\circ \text{ [st. angle]}$$

$$\therefore m(\angle DFE) = 180^\circ - (50^\circ + 65^\circ) = 65^\circ$$

[43]

Const. : Draw \overline{AB}

- $\therefore ABZX$ is a cyclic quad.

$$\therefore m(\angle ABZ) + m(\angle X) = 180^\circ \rightarrow (1) \quad \text{[two opp. angles]}$$

- $\therefore ABLY$ is a cyclic quad.

$$\therefore m(\angle ABZ) = m(\angle Y) \rightarrow (2) \quad \text{[an exterior and interior angle at opp. vertex]}$$

From (1) and (2)

$$\therefore m(\angle Y) + m(\angle X) = 180^\circ \quad \text{[and they are in position of interior supp. angles]}$$

$$\therefore \overline{XZ} \parallel \overline{LY}$$

[44]

- $\therefore \overline{EA}, \overline{EC}$ are two tangent – segment to the small circle

$$\therefore EA = EC \rightarrow (1)$$

- $\therefore \overline{EB}, \overline{ED}$ are two tangent – segment to the big circle

$$\therefore EB = ED \rightarrow (2)$$

By subtracting (1) from (2)

$$\therefore AB = CD$$

[45]

- $\therefore \overline{DA}, \overline{DB}$ are two tangent – segment to circle M

$$\therefore DA = DB$$

$$\therefore m(\angle DBA) = m(\angle DAB) \rightarrow (1)$$

- $\therefore \overline{DA}, \overline{DC}$ are two tangent – segment to circle N

$$\therefore DA = DC$$

$$\therefore m(\angle DCA) = m(\angle DAC) \rightarrow (2)$$

By Adding (1) and (2)

$$\therefore m(\angle DBA) + m(\angle DBA) = m(\angle DAB) + m(\angle DAC)$$

$$\therefore m(\angle DBA) + m(\angle DBA) = m(\angle BAC)$$

$$\therefore m(\angle BAC) = 90^\circ$$

With my best wishes for you

Mr. Michael Gamil