Final Revision (2)

Unit (1) Equations

❶ Find graphically and algebraically the s.s. in $\mathbb{R} \times \mathbb{R}$ of the two equations :

① $3x + 4y = 11$, $2x + y - 4 = 0$

② $2y - 3x = 7$, $2x + -4 + 3y = 0$

❷ Find the s.s. in $\mathbb{R} \times \mathbb{R}$ of the two equations :

① $y - x = -3$, $x^2 + y^2 = 17$

② $x - 2y = 1$, $x^2 - xy = 0$

③ $x - 2y = 1$, $x^2 + y^2 - xy = 19$

❸ Solve the following equation :

① $3x^2 = 5x + 5$ approximating to the nearest two decimals

② $(x - 3)^2 - 5x = 0$ approximating to the nearest one decimals

③ $x - \frac{4}{x} = 1$ given that : $\sqrt{17} \cong 4.12$

❹ ① The sum of two rational numbers is 12 , and three times the smallest number exceeds than twice the greatest number by one , Find the two numbers.

② The length of a rectangle exceeds 3cm. than its width , if twice the length decrease 2cm. than four times its width. Find the length and the width of the rectangle.

③ Since 6 years ago the age of a man was six times his son's age , after ten years the age of this man will be double his son's age. Find the age of both of them.

④ Find the number which formed from two digits , if their sum is 5 and if the two digits are exchanged then the resulting decreases than the original number by 9

Unit (2) Algebraic fractional functions

❶ **Find $n(x)$ in the simplest form showing the domain where :**

① $n(x) = \frac{x^2+2x+4}{x^3-8} + \frac{x^2+x-2}{x^2-4}$

② $n(x) = \frac{x-3}{x^2-7x+12} - \frac{4}{x^2-4x}$

③ $n(x) = \frac{x}{x+1} + \frac{2x^2}{x^3-x}$

④ $n(x) = \frac{3x}{x^2-x-2} + \frac{x-1}{1-x^2}$

⑤ $n(x) = \frac{x^2-3x+2}{x^2-1} \div \frac{3x-15}{x^2-4x-5}$

⑥ $n(x) = \frac{x^3-8}{x^2+x-6} \times \frac{x+3}{x^2+2x+4}$ find : $f(0), f(-3)$

❷ If $n_1(x) = \frac{x^2}{x^3-x^2}$, $n_2(x) = \frac{x^3+x^2+x}{x^4-x}$, Prove that : $n_1 = n_2$

❸ If the set of zeroes of the function f where $f(x) = ax^2+bx+8$ is $\{2,4\}$. Find the value of a and b

❹ If the domain of the function n where $n(x) = \frac{b}{x} + \frac{9}{x+a}$ is $R - \{0,4\}$, $n(5) = 2$ Find the value of a and b

❺ If $n(x) = \frac{x^3+3x^2+2x}{x^2+2x}$, find $n^{-1}(x)$ in the simplest form showing the domain of n^{-1}, then find $n^{-1}(-2)$, $n^{-1}(1)$ if it is possible.

Unit (3) The probability

❶ If : $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, then find $P(A \cup B)$ in the following cases :

(a) $P(A \cap B) = \frac{1}{3}$ (b) A and B are mutually exclusive events.

❷ A card is drawn randomly from 30 identical cards numbered from 1 to 30 , find the probability that the number on the drawn card is :

1- Divisible by 4
2- Multiple of 6
3- Not multiple of 6
4- Divisible by 4 and multiple of 6
5- Divisible by 4 or multiple of 6

❸ If A , B are two events in a random experiment , $P(A) = 0.7$, $P(B) = 0.6$ and $P(A \cap B) = 0.4$, Find the probability of :

1- Occurrence one of them at least.
2- Non-occurrence of the event A

❹ A classroom consists of 40 students , 30 of them play football , 20 play basketball and 15 play football and basketball , if a student is chosen randomly. Find the probability of that this student is playing :

1- Football.
2- One of the two games at least.
3- Not playing any of two games.

Model answer

Unit (1) Equations

❶Find graphically and algebraically the s. s. in $\mathbb{R} \times \mathbb{R}$ of the two equations:

① $\mathbf{3x + 4y = 11}$, $\mathbf{2x + y - 4 = 0}$

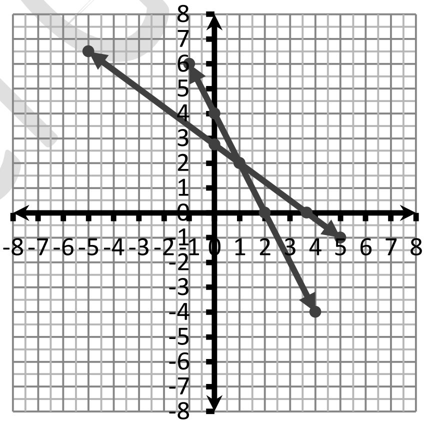
$$\begin{array}{ccccccccc} 3x & + & 4y & = & 11 & \rightarrow & (1) & \times & 1 \\ 2x & + & y & = & 4 & \rightarrow & (2) & \times & -4 \end{array}$$

$$\begin{array}{cccccccc} 3x & + & 4y & = & 11 & \rightarrow & (3) \\ -8x & - & 4y & = & -16 & \rightarrow & (4) \\ \hline -5x & & & = & -5 & & \\ & \therefore & x & = & 1 & & \end{array}$$

By substitution in (2)

$$\begin{array}{ccccc} 2 & + & y & = & 4 \\ & \therefore & y & = & 2 \end{array}$$

$\therefore S.S. = \{(1\,,2)\}$



$\mathbf{3x + 4y = 11}$

x	0	3.7	1
y	2.75	0	2

$\mathbf{2x + y = 4}$

x	0	2	1
y	4	0	2

$\therefore S.S. = \{(1\,,2)\}$

② $\mathbf{2y - 3y = 7}$, $\mathbf{2x + -4 + 3y = 0}$

$$\begin{array}{ccccccccc} -3x & + & 2y & = & 7 & \rightarrow & (1) & \times & 2 \\ 2x & + & 3y & = & 4 & \rightarrow & (2) & \times & 3 \end{array}$$

$$\begin{array}{ccccccc} -6x & + & 4y & = & 14 & \rightarrow & (3) \\ 6x & + & 9y & = & 12 & \rightarrow & (4) \\ \hline & & 13y & = & 26 & & \\ & \therefore & y & = & 2 & & \end{array}$$

By substitution in (2)

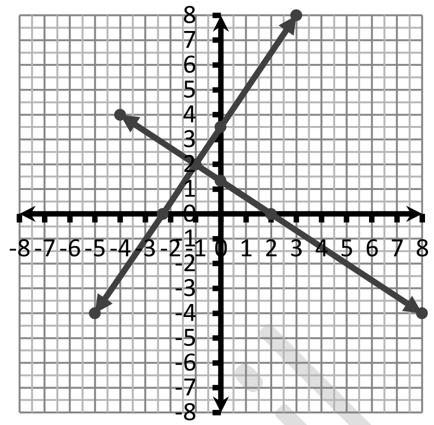
$$\begin{array}{ccccc} 2x & + & 6 & = & 4 \\ & \therefore & x & = & -1 \end{array}$$

$\therefore S.S. = \{(-1\,,2)\}$

$\mathbf{2y - 3x = 7}$ $\mathbf{2x + 3y = 4}$

x	0	-2.3	-1
y	3.5	0	2

x	0	2	-1
y	1.3	0	2



$\therefore S.S. = \{(-1, 2)\}$

❷ Find the s.s. in $\mathbb{R} \times \mathbb{R}$ of the two equations :

① $\mathbf{y - x = -3}$, $\mathbf{x^2 + y^2 = 17}$

$y = -3 + x \quad \rightarrow (1)$

$x^2 + y^2 = 17$

$x^2 + (-3 + x)^2 - 17 = 0$

$x^2 + 9 - 6x + x^2 - 17 = 0$

$2x^2 - 6x - 8 = 0$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x = 4$ $\quad or \quad$ $x = -1$

By substitution in (1)

$y = -3 + (4) = 1$ $\quad or \quad$ $y = -3 + (-1) = -4$

$S.S. = \{(4, 1), (-1, -4)\}$

② $\mathbf{x - 2y = 1}$, $\mathbf{x^2 - xy = 0}$

$x = 1 + 2y \quad \rightarrow (1)$

$x^2 - xy = 0$

$(1 + 2y)^2 - y(1 + 2y) = 0$

$1 + 4y + 4y^2 - y - 2y^2 = 0$

$2y^2 + 3y + 1 = 0$

$(2y + 1)(y + 1) = 0$

$y = -\frac{1}{2}$ $\quad or \quad$ $y = -1$

By substitution in (1)

$x = 1 + 2\left(-\frac{1}{2}\right) = 0$ $\quad or \quad$ $x = 1 + 2(-1) = -1$

$S.S. = \left\{\left(0, -\frac{1}{2}\right), (-1, -1)\right\}$

③ $\boldsymbol{x - 2y = 1}$, $\boldsymbol{x^2 + y^2 - xy = 19}$

$x = 1 + 2y \quad \rightarrow (1)$

$x^2 + y^2 - xy = 19$

$(1 + 2y)^2 + y^2 - y(1 + 2y) - 19 = 0$

$1 + 4y + 4y^2 + y^2 - y - 2y^2 - 19 = 0$

$3y^2 - 3y - 18 = 0$

$y^2 - y - 6 = 0$

$(y - 3)(y + 2) = 0$

$y = 3$ $\quad or \quad$ $y = -2$

By substitution in (1)

$x = 1 + 2(3) = 7$ $\quad or \quad$ $x = 1 + 2(-2) = -3$

$S.S. = \{(7, 3), (-3, -2)\}$

❸ Solve the following equation

① $\boldsymbol{3x^2 = 5x + 5}$ **approximating to the nearest two decimals**

$a = 3 \qquad b = -5 \qquad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4(3)(-5)}}{2(6)} = \frac{5 \pm \sqrt{85}}{6}$$

$x_1 = -0.70 \qquad x_2 = 2.37$

$\boldsymbol{S.S. = \{-0.70, 2.37\}}$

② $\boldsymbol{(x - 3)^2 - 5x = 0}$ **approximating to the nearest one decimals**

$x^2 - 11x + 9 = 0$

$a = 1 \qquad b = -11 \qquad c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{11 \pm \sqrt{121 - 4(1)(9)}}{2(1)} = \frac{11 \pm \sqrt{85}}{2}$$

$x_1 = 0.9 \qquad x_2 = 10.1$

$\boldsymbol{S.S. = \{0.9, 10.1\}}$

③ $\boldsymbol{x - \frac{4}{x} = 1}$ $(\times x)$ **given that :** $\boldsymbol{\sqrt{17} \cong 4.12}$

$x^2 - x - 4 = 0$

$a = 1$ $b = -1$ $c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(1)(-4)}}{2(1)} = \frac{1 \pm \sqrt{17}}{2}$$

$x_1 = \frac{1 + 4.12}{2} = 2.56$ $x_2 = \frac{1 - 4.12}{2} = -1.56$

$\boldsymbol{S.S. = \{2.56 , -1.56\}}$

❹ ①

Let the 1^{st} number $= x$

and the 2^{nd} number $= y$

$x + y = 12$ $\rightarrow (1)$ $\times -3$

$3x - 2y = 1$ $\rightarrow (2)$ $\times 1$

$$\begin{array}{rcrcrcl} -3x & - & 3y & = & -36 & \rightarrow & (3) \\ 3x & - & 2y & = & 1 & \rightarrow & (2) \\ & - & 5y & = & -35 & & \\ & \therefore & y & = & 7 & & \end{array}$$

By substitution in (1)

$$\begin{array}{ccccc} x & + & 7 & = & 12 \\ & \therefore & x & = & 5 \end{array}$$

$\therefore$ Two numbers are **5** and **7**

②

Let the length of the rectangle $= x$

and the width of the rectangle $= y$

$x - y = 3$ $\rightarrow (1)$ $\times 4$

$4y - 2x = 2$ $\rightarrow (2)$ $\times 1$

$$\begin{array}{rcrcrcl} 4x & - & 4y & = & 12 & \rightarrow & (3) \\ -2x & + & 4y & = & 2 & \rightarrow & (2) \\ 2x & & & = & 14 & & \\ & \therefore & x & = & 7 & & \end{array}$$

By substitution in (1)

$$\begin{array}{ccccc} 7 & - & y & = & 3 \\ & \therefore & y & = & 4 \end{array}$$

$\therefore$ Two numbers are **7** and **4**

③ **now**

Let man's age $= x$

and his son's age $= y$

6 years ago

man's age $= x - 6$

and his son's age $= y - 6$

$x - 6 = 6(y - 6)$

$x - 6y = -30 \quad \rightarrow (1)$

After 10 years

man's age $= x + 10$

and his son's age $= y + 10$

$x + 10 = 2(y + 10)$

$x - 2y = 10 \quad \rightarrow (2)$

$x - 6y = -30 \quad \rightarrow (1) \quad \times -1$

$x - 2y = 10 \quad \rightarrow (2) \quad \times 1$

$$\begin{array}{ccccccc} -x & + & 6y & = & 30 & \rightarrow & (3) \\ x & - & 2y & = & 10 & \rightarrow & (2) \\ & & 4y & = & 40 & & \\ & \therefore & y & = & 10 & & \end{array}$$

By substitution in (2)

$$\begin{array}{ccccc} x & - & 20 & = & 10 \\ & \therefore & x & = & 30 \end{array}$$

$\therefore$ Man's age is $\mathbf{30}$ ***years old*** and his son's age is $\mathbf{10}$ ***years old***

④ Let the unit digit $= x$ and the tens digit $= y$

$x + y = 5 \quad \rightarrow (1)$

$(x + 10y) - (y + 10x) = 9$

$-9x + 9y = 9$

$-x + y = 1 \quad \rightarrow (2)$

$$\begin{array}{ccccccc} x & + & y & = & 5 & \rightarrow & (3) \\ -x & + & y & = & 1 & \rightarrow & (2) \\ & & 2y & = & 6 & & \\ & \therefore & y & = & 3 & & \end{array}$$

By substitution in (1)

$$\begin{array}{ccccc} x & + & 3 & = & 5 \\ & \therefore & x & = & 2 \end{array}$$

$\therefore$ the number is $\mathbf{32}$

Unit (2) Algebraic fractional functions

❶ Find $n(x)$ in the simplest form showing the domain where :

① $\boldsymbol{n(x) = \frac{x^2+2x+4}{x^3-8} + \frac{x^2+x-2}{x^2-4}}$

$$n(x) = \frac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \frac{(x+2)(x-1)}{(x+2)(x-2)}$$

$Domain = R - \{2, -2\}$

$$n(x) = \frac{1}{x-2} + \frac{x-1}{x-2} = \frac{1+x-1}{x-2} = \frac{x}{x-2}$$

② $\boldsymbol{n(x) = \frac{x-3}{x^2-7x+12} - \frac{4}{x^2-4x}}$

$$n(x) = \frac{x-3}{(x-3)(x-4)} - \frac{4}{x(x-4)}$$

$Domain = R - \{3,4,0\}$

$$n(x) = \frac{1}{x-4} - \frac{4}{x(x-4)}$$

$$n(x) = \frac{x}{x(x-4)} - \frac{4}{x(x-4)} = \frac{x-4}{x(x-4)} = \frac{1}{x}$$

③ $\boldsymbol{n(x) = \frac{x}{x+1} + \frac{2x^2}{x^3-x}}$

$$n(x) = \frac{x}{x+1} + \frac{2x^2}{x(x^2-1)} = \frac{x}{x+1} + \frac{2x^2}{x(x-1)(x+1)}$$

$Domain = R - \{0,1,-1\}$

$$n(x) = \frac{x}{x+1} + \frac{2x}{(x-1)(x+1)}$$

$$n(x) = \frac{x(x-1)}{(x-1)(x+1)} + \frac{2x}{(x-1)(x+1)}$$

$$n(x) = \frac{x^2-x+2x}{(x-1)(x+1)} = \frac{x^2+x}{(x-1)(x+1)} = \frac{x(x+1)}{(x-1)(x+1)} = \frac{x}{(x-1)}$$

④ $\boldsymbol{n(x) = \frac{3x}{x^2 - x - 2} + \frac{x - 1}{1 - x^2}}$

$$n(x) = \frac{3x}{(x-2)(x+1)} - \frac{x-1}{(x-1)(x+1)}$$

$Domain = R - \{2, -1, 1\}$

$$n(x) = \frac{3x}{(x-2)(x+1)} - \frac{1}{(x+1)}$$

$$n(x) = \frac{3x}{(x-2)(x+1)} - \frac{x-2}{(x-2)(x+1)} = \frac{3x - x + 2}{(x-2)(x+1)}$$

$$n(x) = \frac{2x+2}{(x-2)(x+1)} = \frac{2(x+1)}{(x-2)(x+1)} = \frac{2}{x-2}$$

⑤ $\boldsymbol{n(x) = \frac{x^2 - 3x + 2}{x^2 - 1} \div \frac{3x - 15}{x^2 - 4x - 5}}$

$$n(x) = \frac{(x-2)(x-1)}{(x-1)(x+1)} \div \frac{3(x-5)}{(x-5)(x+1)}$$

$$n(x) = \frac{(x-2)(x-1)}{(x-1)(x+1)} \times \frac{(x-5)(x+1)}{3(x-5)}$$

$Domain = R - \{1, -1, 5\}$

$$n(x) = \frac{x-2}{x+1} \times \frac{x+1}{3} = \frac{x-2}{3}$$

⑥ $\boldsymbol{n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}}$ **find : $\boldsymbol{f(0), f(-3)}$**

$$n(x) = \frac{(x-2)(x^2+2x+4)}{(x+3)(x-2)} \times \frac{x+3}{x^2+2x+4}$$

$Domain = R - \{-3, 2\}$

$n(x) = 1$

$n(0) = 1$

$n(-3)$ is not exist bec. $-3 \notin$ the domain

❷ If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$, Prove that : $n_1 = n_2$

① $n_1(x) = \frac{x^2}{x^3 - x^2}$

$n_1(x) = \frac{x^2}{x^2(x-1)}$

$D_1 = \mathbb{R} - \{0, 1\}$

$n_1(x) = \frac{1}{x-1}$

$n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$

$n_2(x) = \frac{x(x^2 + x + 1)}{x(x^3 - 1)}$

$n_2(x) = \frac{x(x^2 + x + 1)}{x(x-1)(x^2 + x + 1)}$

$D_2 = \mathbb{R} - \{0, 1\}$

$n_2(x) = \frac{1}{x-1}$

$\because$ $D_1 = D_2$ and $n_1(x) = n_2(x)$

$\therefore$ $n_1 = n_2$

❸ If the set of zeroes of the function f where $f(x) = ax^2 + bx + 8$ is $\{2,4\}$.
Find the value of a and b

$\because z(f) = \{2,4\}$

$\therefore f(2) = 0 \text{ and } f(4) = 0$

$f(2) = 4a + 2b + 8 = 0$ $f(4) = 16a + 4b + 8 = 0$

$2a + b = -4$ $4a + b = -2$

$$\begin{array}{rcrcrl} 2a & + & b & = & -4 & \rightarrow (1) \\ -4a & - & b & = & 2 & \rightarrow (2) \\ \hline -2a & & & = & -2 & \\ & \therefore & a & = & 1 & \end{array}$$

By substitution in (1)

$$\begin{array}{rcrcr} 2 & + & b & = & -4 \\ & \therefore & b & = & -6 \end{array}$$

❹ If the domain of the function n where $n(x) = \frac{b}{x} + \frac{9}{x+a}$ is $R - \{0,4\}$, $n(5) = 2$

Find the value of a and b

$\because domain = R - \{0,4\}$

$\therefore a = -4$

$\because n(5) = \frac{b}{5} + \frac{9}{5-4} = 2$

$\therefore \frac{b}{5} + 9 = 2$

$\therefore \frac{b}{5} = 2 - 9$

$\therefore \frac{b}{5} = -7$ $\therefore b = -35$

❺ If $n(x) = \dfrac{x^3 + 3x^2 + 2x}{x^2 + 2x}$, find $n^{-1}(x)$ in the simplest form showing the domain of n^{-1}, then find $n^{-1}(-2)$, $n^{-1}(1)$ if it is possible.

$$n^{-1}(x) = \frac{x(x+2)}{x(x^2+3x+2)} = \frac{x(x+2)}{x(x+2)(x+1)}$$

Domain $= \mathbb{R} - \{0 , -2 , 1\}$

$$n^{-1}(x) = \frac{\cancel{x}\ \cancel{(x+2)}}{\cancel{x}\ \cancel{(x+2)}\ (x+1)} = \frac{1}{x+1}$$

$n^{-1}(-2) =$ does not exist because $-2 \notin domain$

$$n^{-1}(1) = \frac{1}{1+1} = \frac{1}{2}$$

Unit (3) The probability

❶ If : $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, then find $P(A \cup B)$ in the following cases :

(a) $P(A \cap B) = \frac{1}{3}$ (b) A and B are mutually exclusive events.

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{3} = \frac{1}{2}$$

b) $P(A \cup B) = P(A) + P(B)$

$$P(A \cup B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

❷ A card is drawn randomly from 30 identical cards numbered from 1 to 30 , find the probability that the number on the drawn card is :

Let the numbers which divisible by 4= $A = \{4,8,12,16,20,24,28\}$

Let the numbers which multiple of 6= $B = \{6,12,18,24,30\}$

1- $P(A) = \frac{7}{30}$
2- $P(B) = \frac{5}{30} = \frac{1}{6}$
3- $P(B^{\backslash}) = 1 - \frac{1}{6} = \frac{5}{6}$
4- $P(A \cap B) = \frac{2}{30} = \frac{1}{15}$
5- $P(A \cup B) = \frac{10}{30} = \frac{1}{3}$

❸ If A , B are two events in a random experiment , $P(A) = 0.7$, $P(B) = 0.6$ and $P(A \cap B) = 0.4$, Find the probability of :

1- Occurrence one of them at least $= P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.6 - 0.4 = 0.9$

2- Non-occurrence of the event A= $P(A^\backslash)$

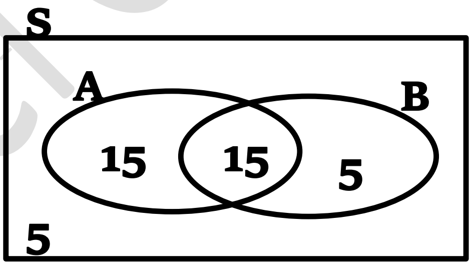
$P(A^\backslash) = 1 - P(A) = 1 - 0.7 = 0.3$

❹ A classroom consists of 40 students , 30 of them play football , 20 play basketball and 15 play football and basketball , if a student is chosen randomly. Find the probability of that this student is playing :

1- Football.
2- One of the two games at least.
3- Not playing any of two games.

Football $= A$

basketball $= B$



① $P(A) = \frac{n(A)}{n(s)} = \frac{30}{40} = \frac{3}{4}$

② $P(A \cup B) = \frac{n(A \cup B)}{n(s)} = \frac{35}{40} = \frac{7}{8}$

③ $P(A \cup B)^\backslash = 1 - P(A \cup B) = 1 - \frac{7}{8} = \frac{1}{8}$

With all my best wishes for you
Mr Michael Gamil